

Lesson 28. Lagrange Multipliers

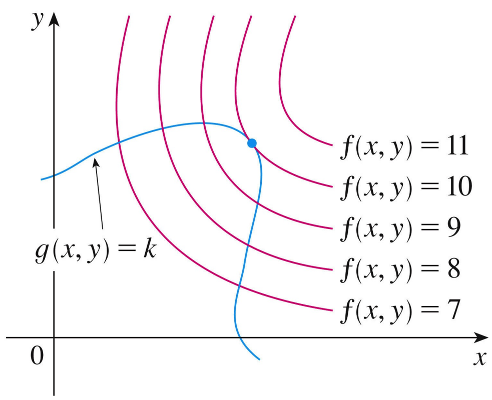
1 Today...

- Optimization with one equality constraint

$$\begin{aligned} \text{minimize or maximize} \quad & f(x,y) \\ \text{subject to} \quad & g(x,y) = k \end{aligned}$$

2 Lagrange multipliers

- Convention: "maximum" and "minimum" refer to "absolute maximum" and "absolute minimum" respectively
- Idea:



  - Maxima and minima occur when the level curves of $f(x,y)$ and the constraint $g(x,y)$ have a common tangent line
  - In other words, the gradients of f and g are parallel:

- **Method of Lagrange multipliers for optimization with one equality constraint**
  - To find the maximum and minimum values of $f(x,y)$ subject to the constraint $g(x,y) = k$:
    1. Find all values of x, y, λ such that

       or equivalently

    2. Evaluate f at all the points (x,y) you found in Step 1.
       - Largest of these values = maximum value of f
       - Smallest of these values = minimum value of f
  - (Assumes extreme values exist and $\nabla g \neq \vec{0}$ on the curve $g(x,y) = k$)
  - Works in a similar way for solving

$$\begin{aligned} \text{minimize or maximize} \quad & f(x,y,z) \\ \text{subject to} \quad & g(x,y,z) = k \end{aligned}$$

Example 1. Find the absolute maximum and minimum of $f(x, y) = y^2 - x^2$ on the ellipse $x^2 + 4y^2 = 4$.

Example 2. Find three positive numbers whose sum is 90 and whose product is a maximum.

Example 3. A rectangular box is to be made from 100 m^2 of cardboard. Find the maximum volume of such a box.